Evaluate the following integrals. If a definite integral cannot be evaluated using the Fundamental Theorem of Calculus, write "FTC DOES NOT APPLY", and explain why not with proper justification.

SCORE: _____ / 55 PTS

[a] $\displaystyle\int_{-1}^{2} \frac{1-x}{\sqrt{5+6x}}\,dx$

3 FTC DOES NOT APPLY —

3 DISCONTINUOUS (DOES NOT EXIST) @ $x \le -\frac{5}{6}$

4

[b] $\displaystyle\int_{-3}^{3} \frac{7t^2}{(4-t^2)^5}\,dt$

3 FTC DOES NOT APPLY —

3 DISCONTINUOUS @ $t = \pm 2$

4

[c] $\displaystyle\int \frac{5e^{2y}}{\sqrt{1-e^{4y}}}\,dy$

4 $u = e^{2y}$

$du = 2e^{2y}\,dy$

$\frac{5}{2}du = 5e^{2y}\,dy$

5 $\displaystyle\int \frac{\frac{5}{2}du}{\sqrt{1-u^2}}$

$= \frac{5}{2}\sin^{-1} u + C$ 5

$= \frac{5}{2}\sin^{-1} e^{2y} + C$ 2 4

[d] $\displaystyle\int_{-\pi}^{\pi} x^5 \cos x\,dx$

4 $(-x)^5 \cos(-x) = -x^5 \cos x$

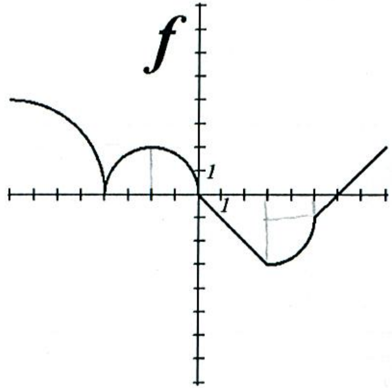
4 ODD, CONTINUOUS 3

INTEGRAL $= 0$

4

The graph of $f(t)$ is shown on the right. **NOTE: The graph consists of a quarter circle, a semi-circle, a line segment, another quarter circle, and another line segment.**

SCORE: _____ / 20 PTS



Let $g(x) = \int_5^x f(t)\,dt$.

[a] Find $g(-2)$.

$$\int_5^{-2} f(t)dt = -\underbrace{\int_{-2}^{5} f(t)dt}_{2} = -\Big[\overbrace{\tfrac{1}{4}\pi(2)^2}^{1} - \overbrace{\tfrac{1}{2}(3)(3)}^{1} \underbrace{-\tfrac{1}{4}\pi(2)^2}_{1} \underbrace{-2}_{1}\Big] = \underbrace{\frac{13}{2}}_{1}$$

[b] Find $g'(-4)$.

$1\tfrac{1}{2}$ $\underbrace{f(-4)}$ $= \underbrace{0}$ $1\tfrac{1}{2}$

[c] Find $g''(2)$.

2 $\underbrace{f'(2)}$ $= \underbrace{-1}$ 2

[d] Find the $x-$coordinates of all local maxima of g.

$\underbrace{g'(x) = f(x) \text{ CHANGES FROM POSITIVE TO NEGATIVE}}$ 3

@ $\underbrace{x = 0}$ 3

Find $\frac{d}{dx}(\tanh^{-1} x^2 + \operatorname{sech} 3x)$.

SCORE: _____ / 15 PTS

You may use any hyperbolic identities or the derivatives of any hyperbolic functions without proving them.

$$\underbrace{\frac{1}{1-(x^2)^2}}_{5} \cdot \underbrace{2x}_{3} + \underbrace{(-\operatorname{sech} 3x \tanh 3x)}_{4}\underbrace{(3)}_{3}$$

$$= \frac{2x}{1-x^4} - 3\operatorname{sech} 3x \tanh 3x$$

Find $\lim_{n\to\infty} \frac{4}{n}\sum_{i=1}^{n} \frac{1}{3+\frac{4i}{n}}$ by finding the corresponding definite integral, and evaluating that integral. **SCORE: _____ / 15 PTS**

$$\int_{\underbrace{3}_{3}}^{7} \underbrace{\frac{1}{x}}_{3} dx = \underbrace{\ln|x|}_{3} \Big|_3^7$$

$$= \ln 7 - \ln 3$$

$$= \underbrace{\ln \frac{7}{3}}_{3}$$

$a + i\Delta x = 3 + \frac{4i}{n}$

$a = 3$, $\Delta x = \frac{4}{n} = \frac{b-a}{n}$

$\frac{4}{n} = \frac{b-3}{n}$

$b = 7$

$f(a+i\Delta x) = f(3+\frac{4i}{n}) = \frac{1}{3+\frac{4i}{n}}$

$f(x) = \frac{1}{x}$

The velocity of a car at time t hours after 1 pm is given by $v(t) = t^2 - 3t - 4$ miles per hour. **SCORE: _____ / 15 PTS**

Find the total distance travelled by, **NOT THE DISPLACEMENT OF**, the car from 2 pm to 5 pm.

$t=1$ $\quad$ $t=4$

$$\int_1^4 |v(t)|\,dt = \int_1^4 |t^2-3t-4|\,dt$$

$$t^2-3t-4 = (t-4)(t+1)$$

$v(t)>0$ $\quad$ <0 $\quad$ >0

-1 $\quad$ 4

(1) (5)

$$= \int_1^4 (-t^2+3t+4)\,dt$$

(1) (4)

$$= \left(-\tfrac{1}{3}t^3 + \tfrac{3}{2}t^2 + 4t\right)\Big|_1^4$$

$$= -\tfrac{1}{3}(64-1) + \tfrac{3}{2}(16-1) + 4(4-1)$$

$$= -21 + \tfrac{45}{2} + 12 = \tfrac{27}{2} \text{ miles}$$

(1) (3)

Prove that $\frac{\pi}{2} \le \int_1^{\sqrt{3}} x^3 \tan^{-1} x \, dx \le \frac{2\pi}{3}$.

SCORE: _____ / 15 PTS

ON $[1, \sqrt{3}]$, 5 $\underbrace{\frac{\pi}{4} \le \tan^{-1} x \le \frac{\pi}{3}}$

$$\frac{\pi}{4} x^3 \le x^3 \tan^{-1} x \le \frac{\pi}{3} x^3$$

4 $\underbrace{\int_1^{\sqrt{3}} \frac{\pi}{4} x^3 \, dx \le \int_1^{\sqrt{3}} x^3 \tan^{-1} x \, dx \le \int_1^{\sqrt{3}} \frac{\pi}{3} x^3 \, dx}$

$$\frac{\pi}{2} = \underbrace{2\left(\frac{\pi}{4}\right) \le \int_1^{\sqrt{3}} x^3 \tan^{-1} x \, dx \le 2\left(\frac{\pi}{3}\right)} = \frac{2\pi}{3}$$

4

$$\int_1^{\sqrt{3}} x^3 \, dx$$

2 $= \underbrace{\frac{1}{4} x^4} \Big|_1^{\sqrt{3}}$

$$= \frac{1}{4}(9-1) = 2$$